Nb_3Sn Short Sample Calculations and Strain Effects

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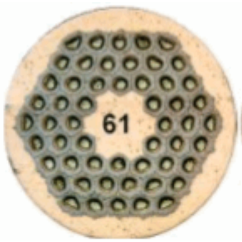
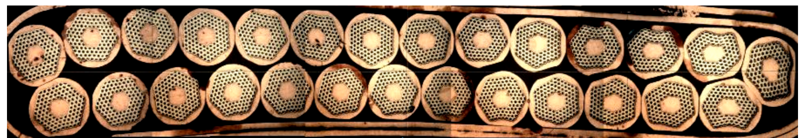
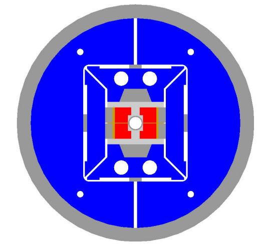
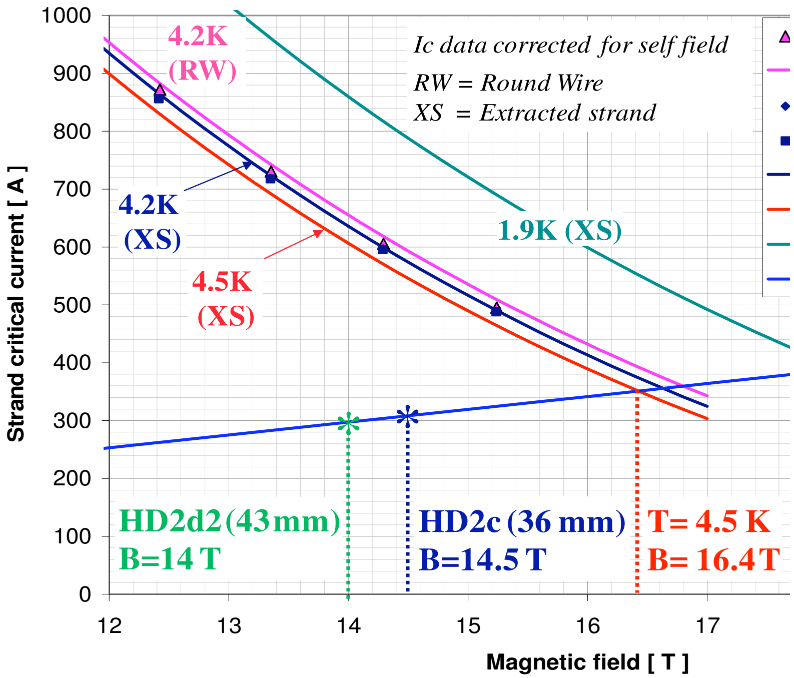
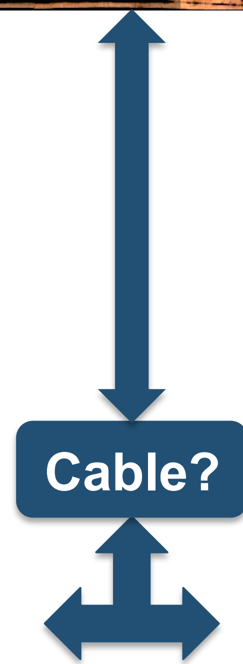
Example: LBNL – HD2

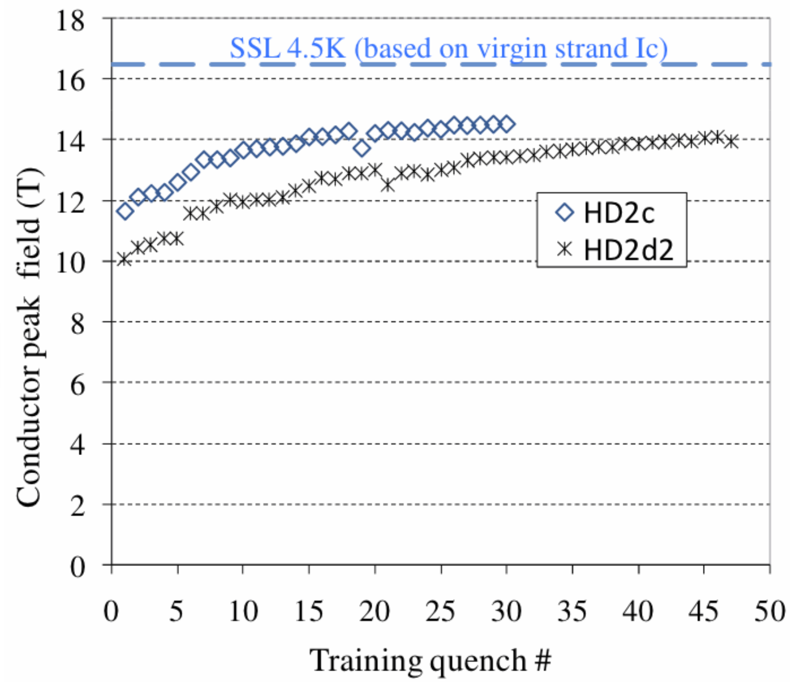












Performance is mainly judged from extracted strand data: What is $I_c(B,T,\varepsilon)$?

Parameterizations: $I_c(B,T,\varepsilon)$

I_c scaling vs. B, T, ε

- Separation of parameters:

$$F_P = J_c(B,T,\varepsilon) \times B$$

$$= C\, g(\varepsilon)\, h(t)\, f_P(b)$$

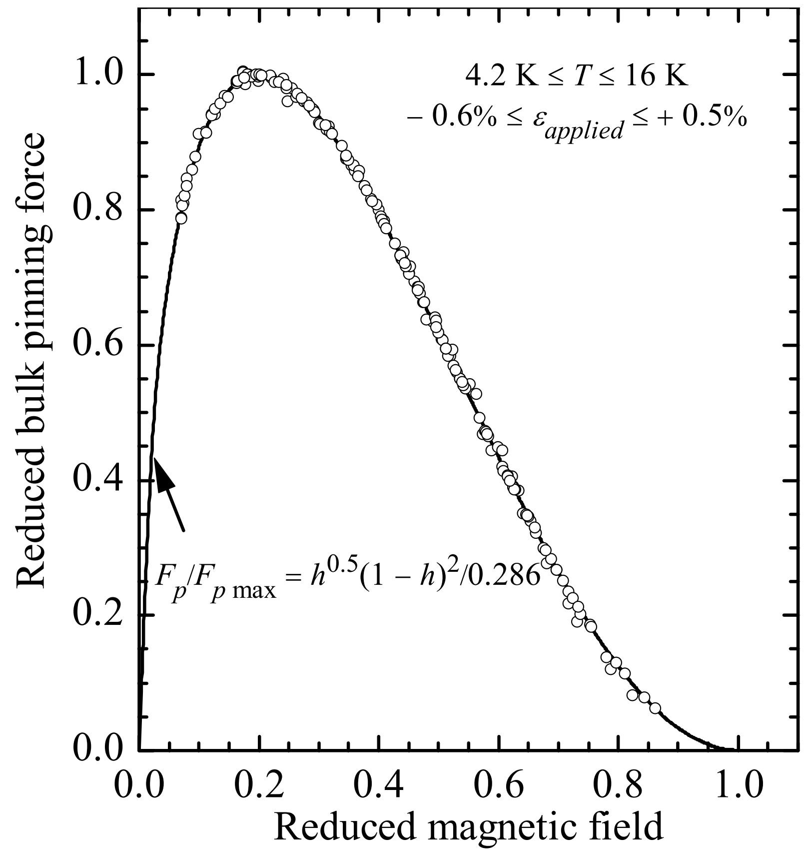
- $t = T/T_c$, $b = B/B_{c2}$
  - T_c and B_{c2} are *effective* values

- $g(\varepsilon)$ = some function of strain
  - Common: $g(\varepsilon) = s(\varepsilon) \equiv B_{c2}(\varepsilon)/B_{c2m}$

$$F_P \propto b^{0.5}(1 - b)^2$$

Magnetic field dependence

- $F_P = C\, g(\varepsilon)\, h(t)\, f_P(b)$

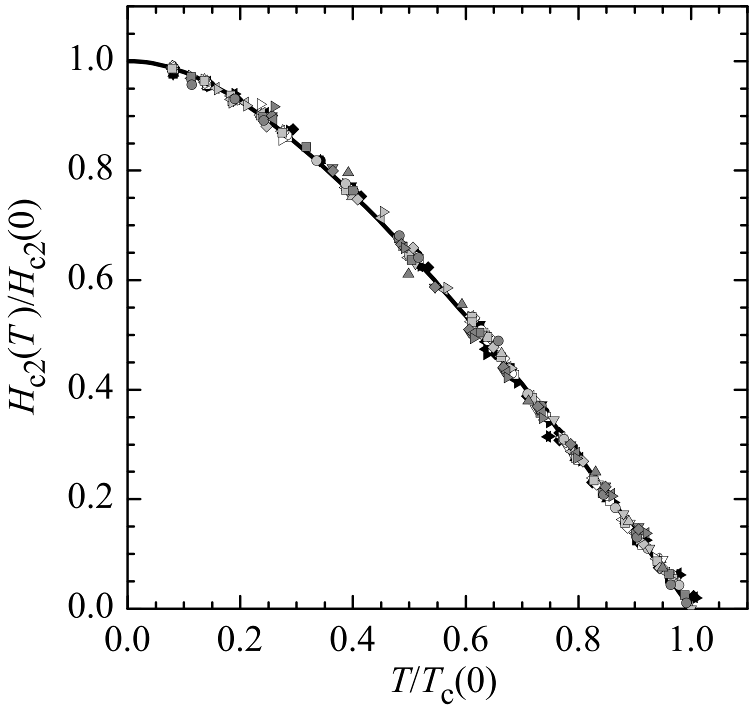


Godeke *et al.*, *Supercond. Sci. Techn.* **19**, R100 (2006)

Parameterizations: $I_c(B,T,\varepsilon)$

Temperature dependence

- $F_P = C\, g(\varepsilon)\, h(t)\, f_P(b)$



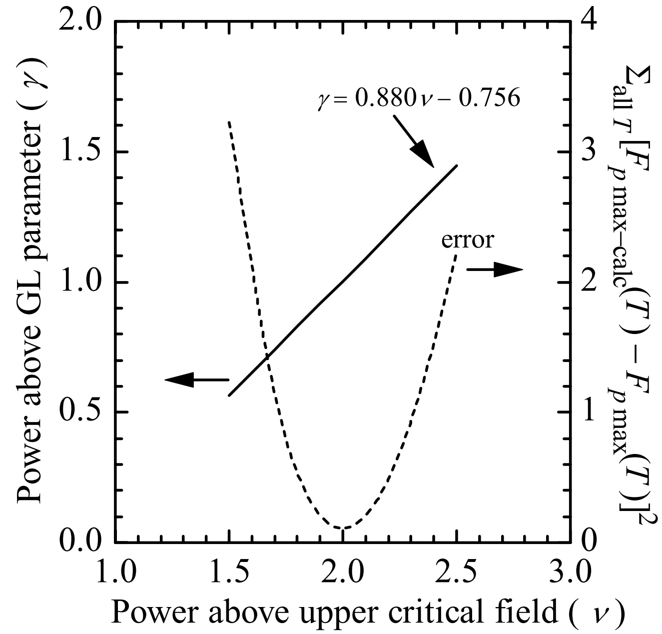
Godeke *et al.*, *J. Appl. Phys.* **97**, 093909 (2005)

- $B_{c2}(t)/B_{c2}(0) = \mathrm{MDG}(t) \approx 1 - t^{1.52}$

$$F_P \propto \mathrm{MDG}(t)\,(1 - t^2)$$

General form

- $h(t) \propto B_{c2}(t)^{\nu} / \kappa_1(t)^{\gamma} = B_{c2}(t)^{(\nu-\gamma)}\, B_c(t)^{\gamma}$



Godeke *et al.*, *Supercond. Sci. Techn.* **19**, R100 (2006)

- $h(t) \propto B_{c2}(t)\, B_c(t)$

  - $h(t) = \mathrm{MDG}(t)(1 - t^2)$
    $\approx (1 - t^{1.52})(1 - t^2)$

Parameterizations: $I_c(B,T,\varepsilon)$

Resulting relations

- $F_P = C\, g(\varepsilon)\, h(t)\, f_P(b)$
  - $I_c(B,T,\varepsilon)\, B = C\, s(\varepsilon)\, \mathrm{MDG}(t)\, (1 - t^2)\, b^{0.5}(1 - b)^2$

Mathematical re-hash: Godeke, Mentink, *et al.*, *IEEE Trans. Appl. Supercond.* **19**, 2610 (2009)

$$\boxed{I_c(B,T,\varepsilon) = C'\, (1 - t^2)\, b^{-0.5}(1 - b)^2}$$

with

- $t = T/T_c(0,\varepsilon)$
- $T_c(0,\varepsilon) = T_{cm}(0)\, s(\varepsilon)^{1/3}$

$b = B/B_{c2}(T,\varepsilon)$

$B_{c2}(T,\varepsilon) = B_{c2m}(0)\, \mathrm{MDG}(t)\, s(\varepsilon)$

$\left(= B_{c2m}(0)\, (1 - t^{1.52})\, s(\varepsilon)\right)$

To what extend does the community agree?

Independent, objective comparison of alternatives

	$g(\varepsilon)$	$h(t)$	$f_P(b)$	$s(\varepsilon)$	# Par.
Ekin	$[s(\varepsilon)]^{v}$ (†)	$(1-t^{v})^{\eta}$	$b^{p}(1-b)^{q}$	$s(\varepsilon)=1-a\lvert\varepsilon\rvert^{1.7}$	10 – 14
Summers et al.	$s(\varepsilon)$	$\left[1-0.31\,t^{2}(1-1.77\ln(t))\right]^{0.5}(1-t^{2})^{2.5}$	$b^{0.5}(1-b)^{2}$	$s(\varepsilon)=1-a\lvert\varepsilon\rvert^{1.7}$	4
Durham	$[s(\varepsilon)]^{\frac{w(n-2)+2+}{w}}$	$(1-t^{v})^{n-2}(1-t^{2})^{2}$	$b^{p}(1-b)^{q}$	$s(\varepsilon)=1+c_2\varepsilon^2+c_3\varepsilon^3+c_4\varepsilon^4$	13 – 17
Twente.	$s(\varepsilon)$	$(1-t^{1.52})(1-t^{2})$	$b^{0.5}(1-b)^{2}$	$s(\varepsilon)=1+\frac{C_{a1}\left(\sqrt{\varepsilon_{sh}^2+\varepsilon_{0,a}^2}-\sqrt{(\varepsilon-\varepsilon_{sh})^2+\varepsilon_{0,a}^2}\right)-C_{a2}\varepsilon}{1-C_{a1}\varepsilon_{0,a}}$ $\varepsilon_{sh}=\frac{C_{a2}\varepsilon_{0,a}}{\sqrt{C_{a1}^2-C_{a2}^2}}$	7
Markiewicz	-	-		$s(\varepsilon)=\frac{1}{1+c_2\varepsilon^2+c_3\varepsilon^3+c_4\varepsilon^4}$	
Oh and Kim	$[s_B(\varepsilon)]^{2.5}\,[k(T,\varepsilon)]^{0.5}\,(1-t^{2.17})^{2.5}$ (‡)		$b^{0.5}(1-b)^{2}$	$s_B(\varepsilon)=1-\beta\lvert\varepsilon\rvert^{1.7}$ (*)	9 – 12
ITER-2008	$s(\varepsilon)$	$(1-t^{1.52})(1-t^{2})$	$b^{p}(1-b)^{q}$	$s(\varepsilon)=1+\frac{C_{a1}\left(\sqrt{\varepsilon_{sh}^2+\varepsilon_{0,a}^2}-\sqrt{(\varepsilon-\varepsilon_{sh})^2+\varepsilon_{0,a}^2}\right)-C_{a2}\varepsilon}{1-C_{a1}\varepsilon_{0,a}}$ $\varepsilon_{sh}=\frac{C_{a2}\varepsilon_{0,a}}{\sqrt{C_{a1}^2-C_{a2}^2}}$	9

Bottura and Bordini., *IEEE Trans. Appl. Supercond.* **19**, 1521 (2009)

Achievable STD across entire space on wires = 3 ÷ 5%

Selection was made. What about the strain function $s(\varepsilon)$?

What does strain do?

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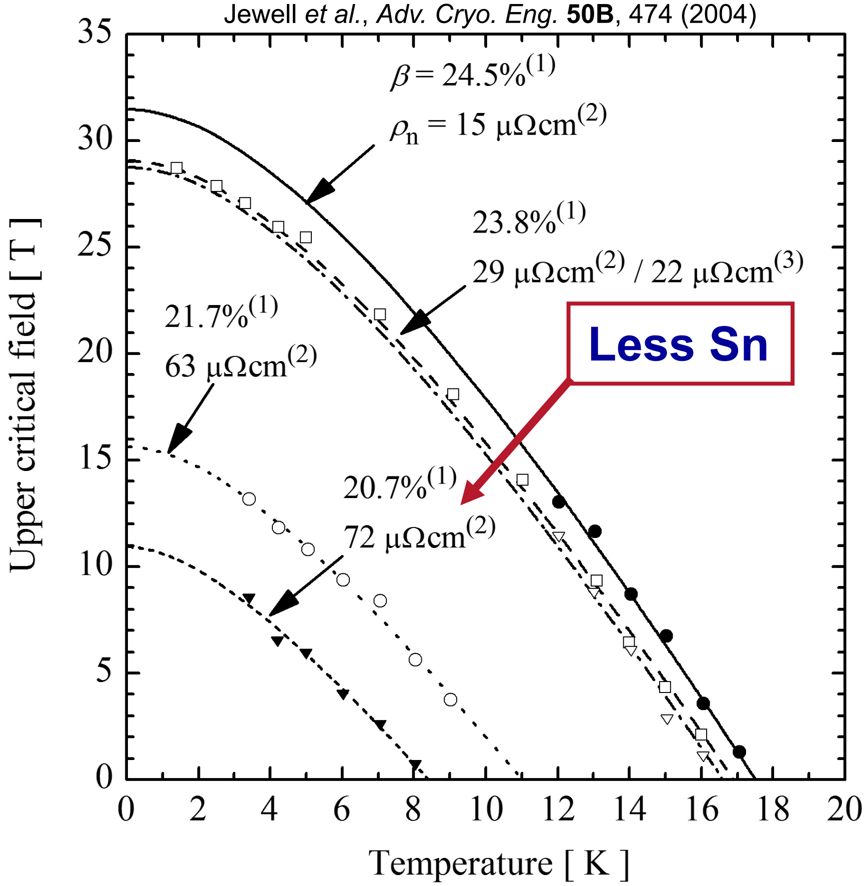
Composition effects on $B_{c2}(T)$

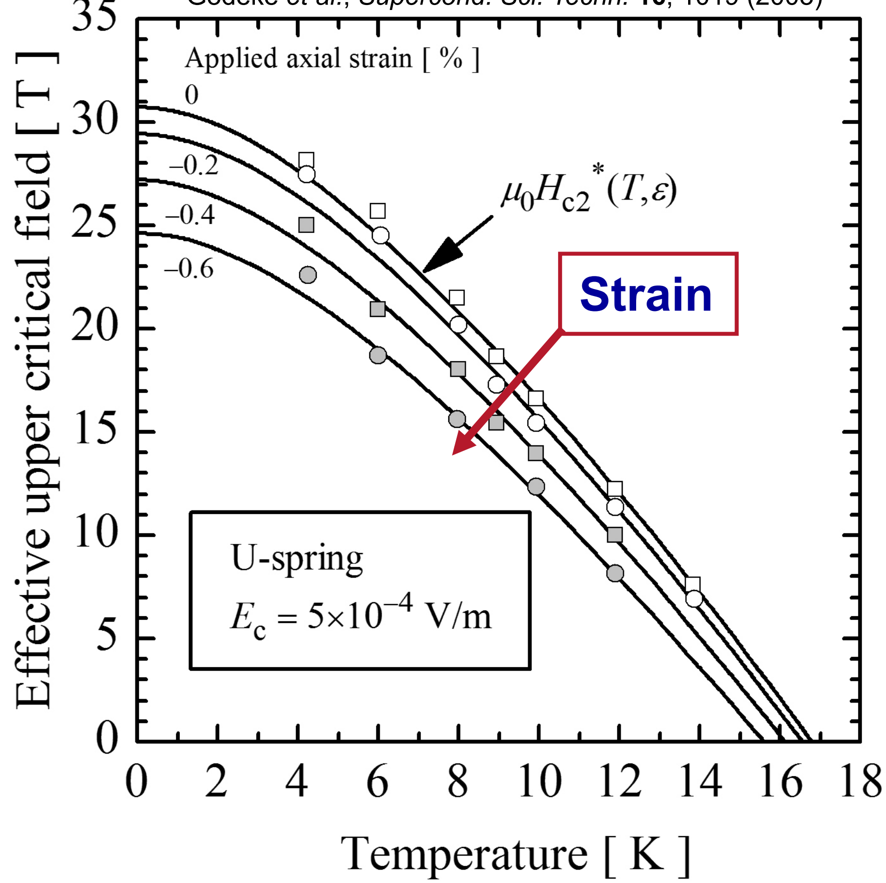
Jewell *et al.*, *Adv. Cryo. Eng.* **50B**, 474 (2004)



- Leads to averaged, effective $B_{c2}(T)$

Strain effects on $B_{c2}(T)$

Godeke *et al.*, *Supercond. Sci. Techn.* **16**, 1019 (2003)



Why does strain affect $B_{c2}(T)$?

Strain modifies

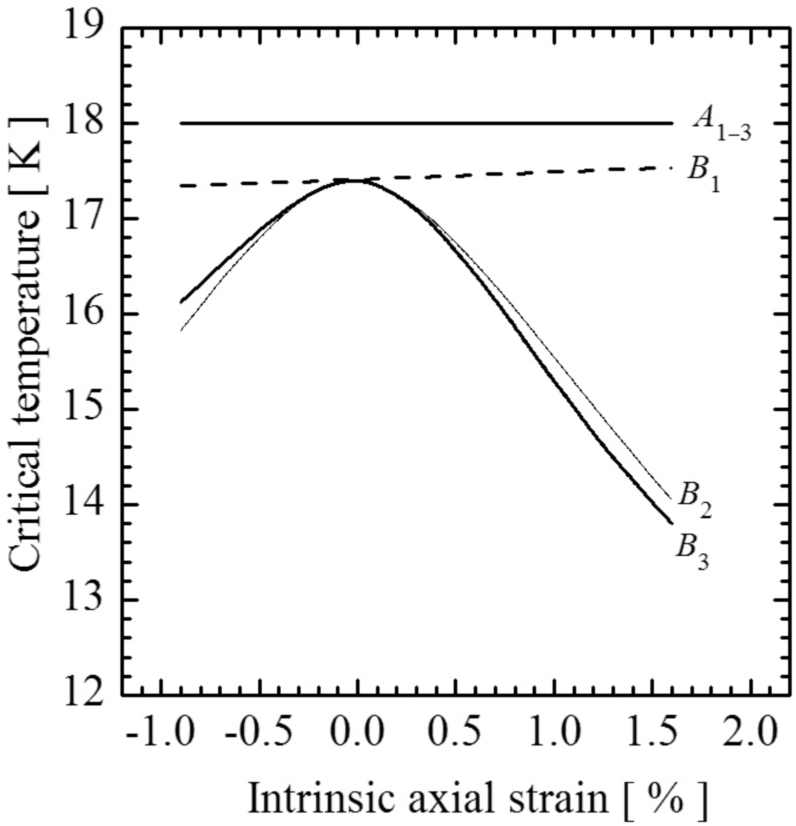
- Lattice vibration modes (phonons)
- Electron-phonon interaction spectrum

Markiewicz, *Cryogenics* **44**, 676 and 895 (2004)
Markiewicz, Trans. Appl. Supercond. **15**, 3368 (2005)

$$\lambda_{\mathrm{ep}} = 2\int \frac{\alpha^2(\omega)F(\omega)}{\omega} d\omega$$

$$\lambda_{\mathrm{eff}} = \frac{\left(\lambda_{\mathrm{ep}} - \mu^*\right)}{\left(1 + 2\mu^* + 1.5\lambda_{\mathrm{ep}}\mu^* e^{-0.28\lambda_{\mathrm{ep}}}\right)}$$

$$T_{\mathrm{c}} = \frac{0.25\langle\omega^2\rangle^{\frac{1}{2}}}{\left(e^{2/\lambda_{\mathrm{eff}}} - 1\right)^{\frac{1}{2}}} \qquad \mu_0 H_{\mathrm{c2}} = \ldots?$$



Strain free value

Hydrostatic (1st inv)

Symmetric (2nd inv) non-hydrostatic

Asymmetric (3rd inv) non-hydrostatic

Can this be simplified while retaining physics and 3D?

A 3D based, axial $s(\varepsilon)$ for wires

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From strain energy function

- All invariants

$$\frac{H^*_{c2}(\boldsymbol{\epsilon})}{H^*_{c2}(\boldsymbol{\epsilon}=\mathbf{0})} = (1 - C_{\mathrm{h}} I_1)\left(1 - C_{\mathrm{d},1}\sqrt{J_2} - C_{\mathrm{d},2} J_3\right)$$

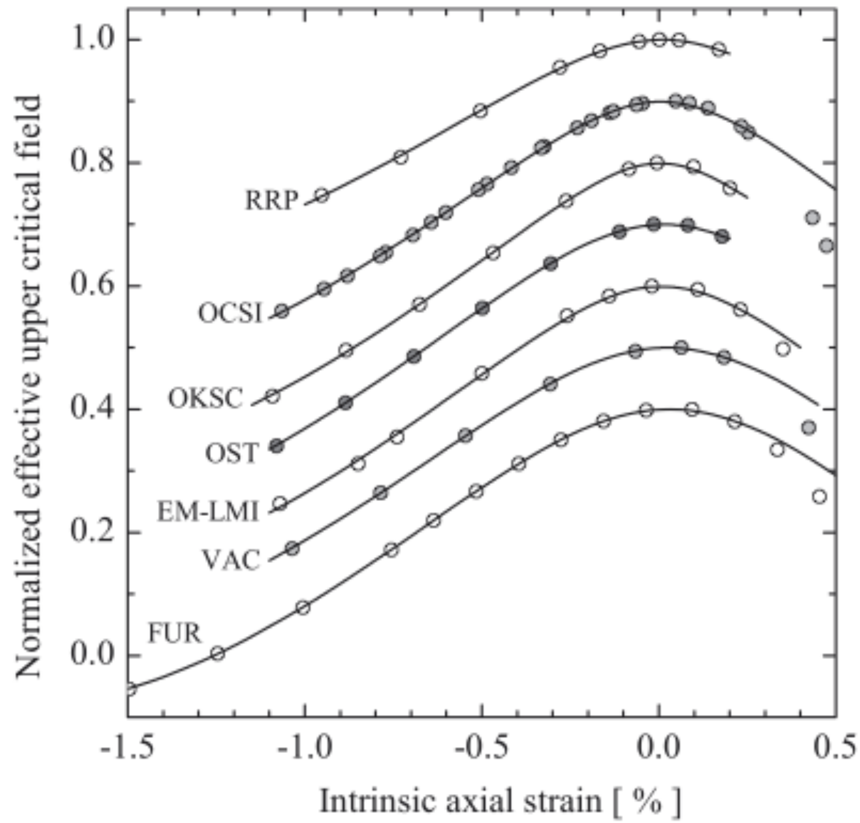
In axial form for wires:

$$s(\varepsilon_{\mathrm{I}}) = \frac{H^*_{c2}(\varepsilon_{\mathrm{I}})}{H^*_{c2}(\varepsilon_{\mathrm{I}}=0)}$$
$$= \frac{1 - C_{\mathrm{a},1}\sqrt{(\varepsilon_{\mathrm{I}})^2 + (\varepsilon_{0,\mathrm{a}})^2} - C_{\mathrm{a},2}\left((\varepsilon_{\mathrm{I}})^3 - 3(\varepsilon_{0,\mathrm{a}})^2\varepsilon_{\mathrm{I}}\right)}{1 - C_{\mathrm{a},1}\varepsilon_{0,\mathrm{a}}}$$

- Without loss of accuracy:
  - $C_{a2} = 1034 \times C_{a1}$
  - 3 fit parameters: C_{a1}, $\varepsilon_{0,a}$, ε_m
  - 3D form to be validated

Comparison to measurement

Arbelaez *et al.*, *Supercond. Sci. Techn.* **22**, 025005 (2009)



Why the emphasis on 3D?

Multi-scale model development

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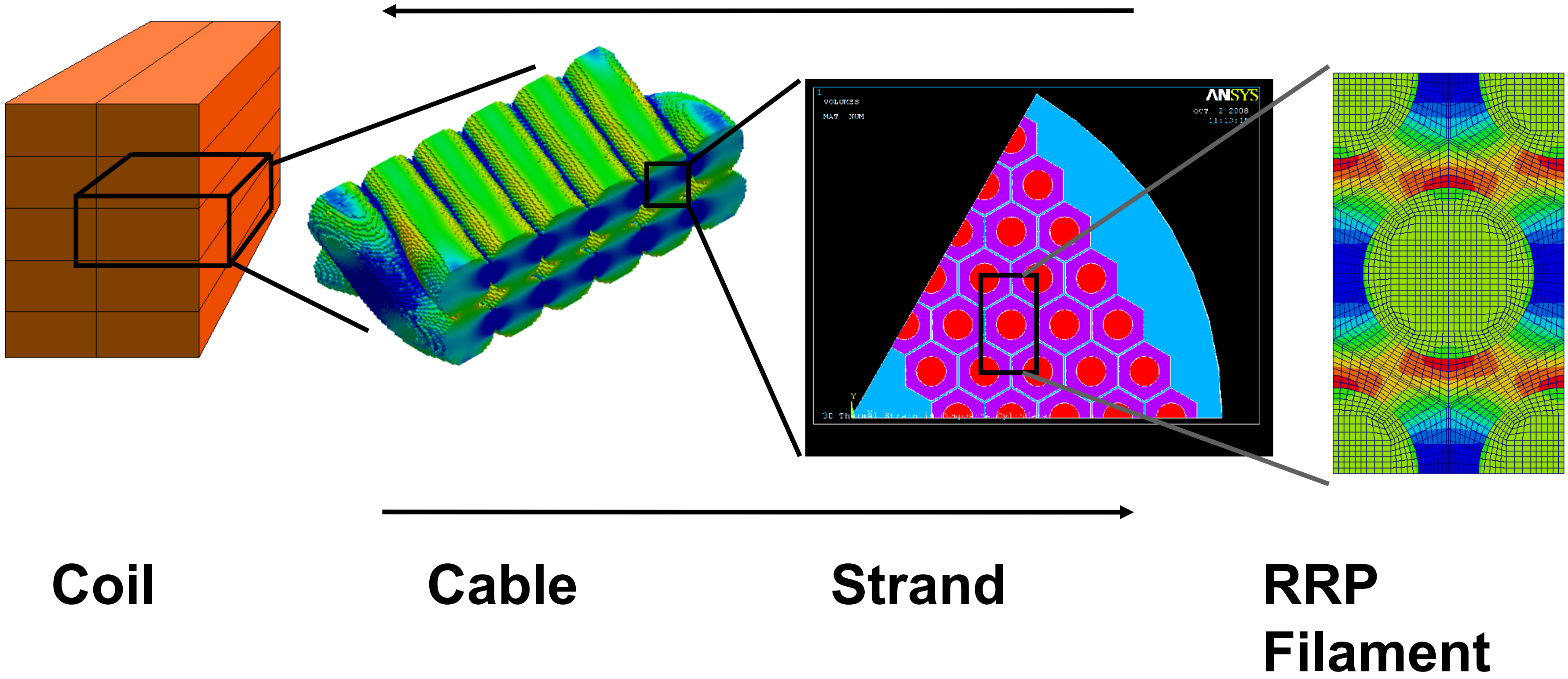
3D strain state at the filaments for applied macro-scale loads

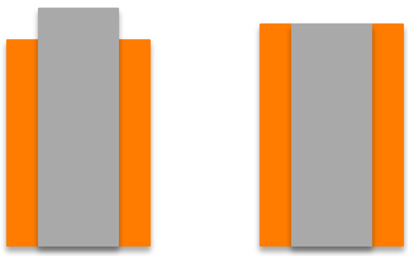
- Arbelaez *et al.*, EUCAS 2009



But, for now, only 1D is feasible

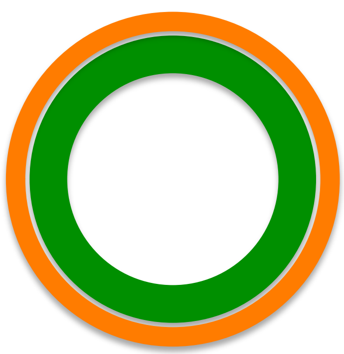
Ballpark axial strain states in Nb_3Sn fractions

Wire: Nb_3Sn in Cu



$\varepsilon_a = -0.35\%$

Wire on ITER barrel



Ti-6Al-4V: $\varepsilon_a = -0.2\%$
SS: $\varepsilon_a = -0.3\%$

Ghosh, *BNL report* MDN-657-39 (2009)

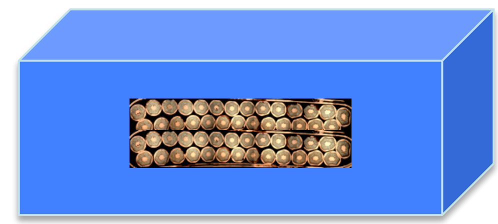
THERMAL CONTRACTION FROM R.T. TO 4.2 K (OR10 K)

Copper:	-0.30%
Stainless Steel	-0.27%
Ti-6Al-4V	-0.15%
G-10	-0.28%
Nb3Sn	-0.15%
Inconel 600	-0.27%
Composite Nb3Sn strand:	-0.15% to -0.29%
Iron	-0.2%

Ghosh, *BNL report* MDN-657-39 (2009)

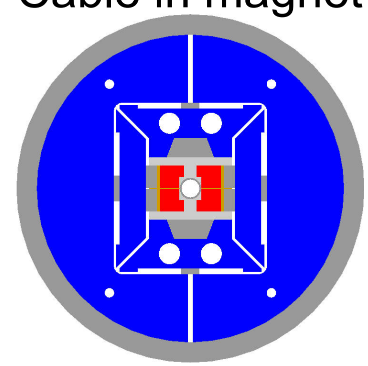
Barrel Material	ΔIc/Ic	Δε
Ti-Al-V	0.00	0
G-10	0.10	-0.10%
SS-304	0.08	-0.07%
SS- Soldered	0.19	-0.15%

Cable in SS holder



$\varepsilon_a = -0.3\%$

Cable in magnet



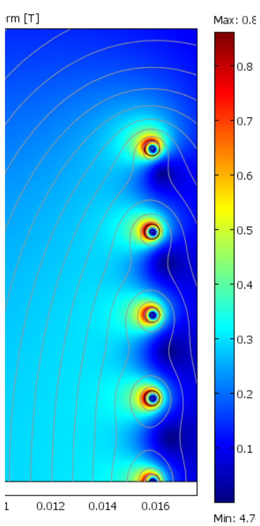
Loaded iron enclosure with Ti-6Al-4V poles
$\varepsilon_a = -0.2\%$?

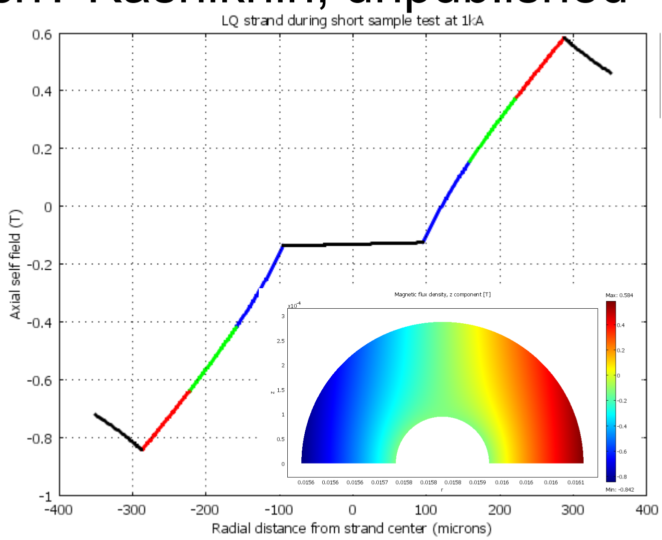
I_c results for extracted strands on Ti-alloy barrels

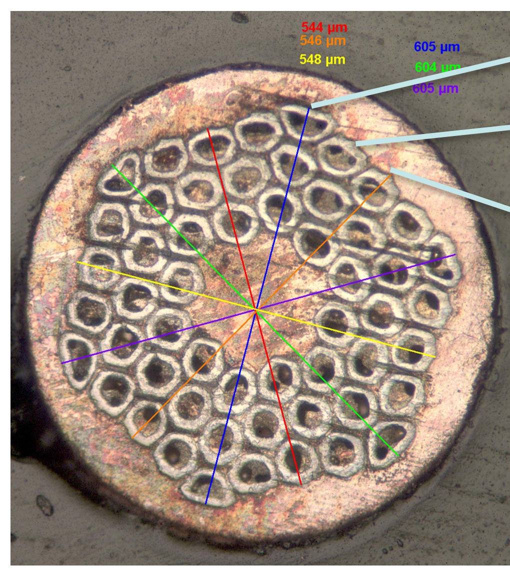
- Self-field correction required
  - Why?

Applied field [T]	Measured I_c [A]	Total field [T]
12.0	571	12.32
11.0	693	11.39
10.0	836	10.47

  - How much? Kashikhin, unpublished







0.584 mT/A

0.555 mT/A

0.483 mT/A

Parameterization of XS data

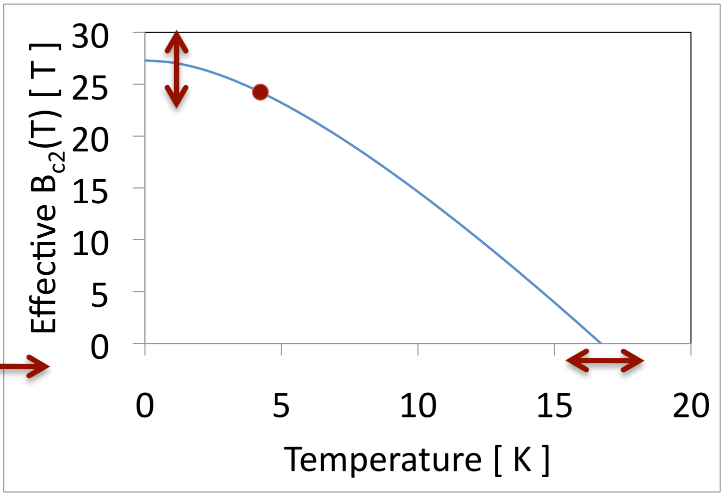
Bapplied [T]	Btotal [T]	Ic-meas	Ic-calc
11.5	11.85	628	625
11.0	11.39	693	691
10.5	10.92	761	762
10.0	10.47	836	838
9.5	10.01	916	919
9.0	9.56	1003	1007
8.0	8.67	1205	1199

Ti-6Al-4V barrel

$\varepsilon_a = -0.2\%$

$T = 4.23$ K

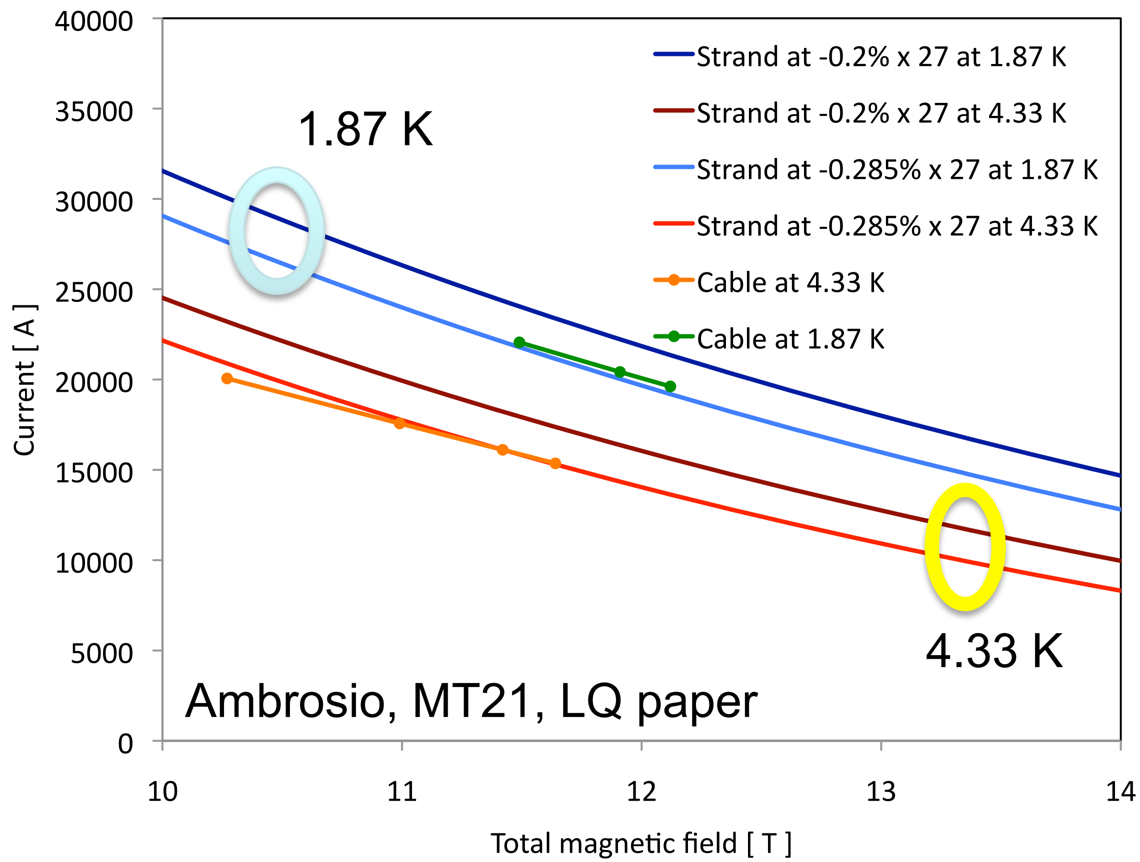
Parameter	Value	Determined
C_{a1} [T]	48.6	From strain data
$\varepsilon_{0,a}$ [%]	0.222	From strain data
$B_{c2m}(0)$ [T]	27.3	Fitted to data
$T_{cm}(0)$ [K]	16.7	Chosen if unknown
C [AT]	54809	Fitted to data



From XS to cable

Comparison of strand scaling and SF corrected cable data

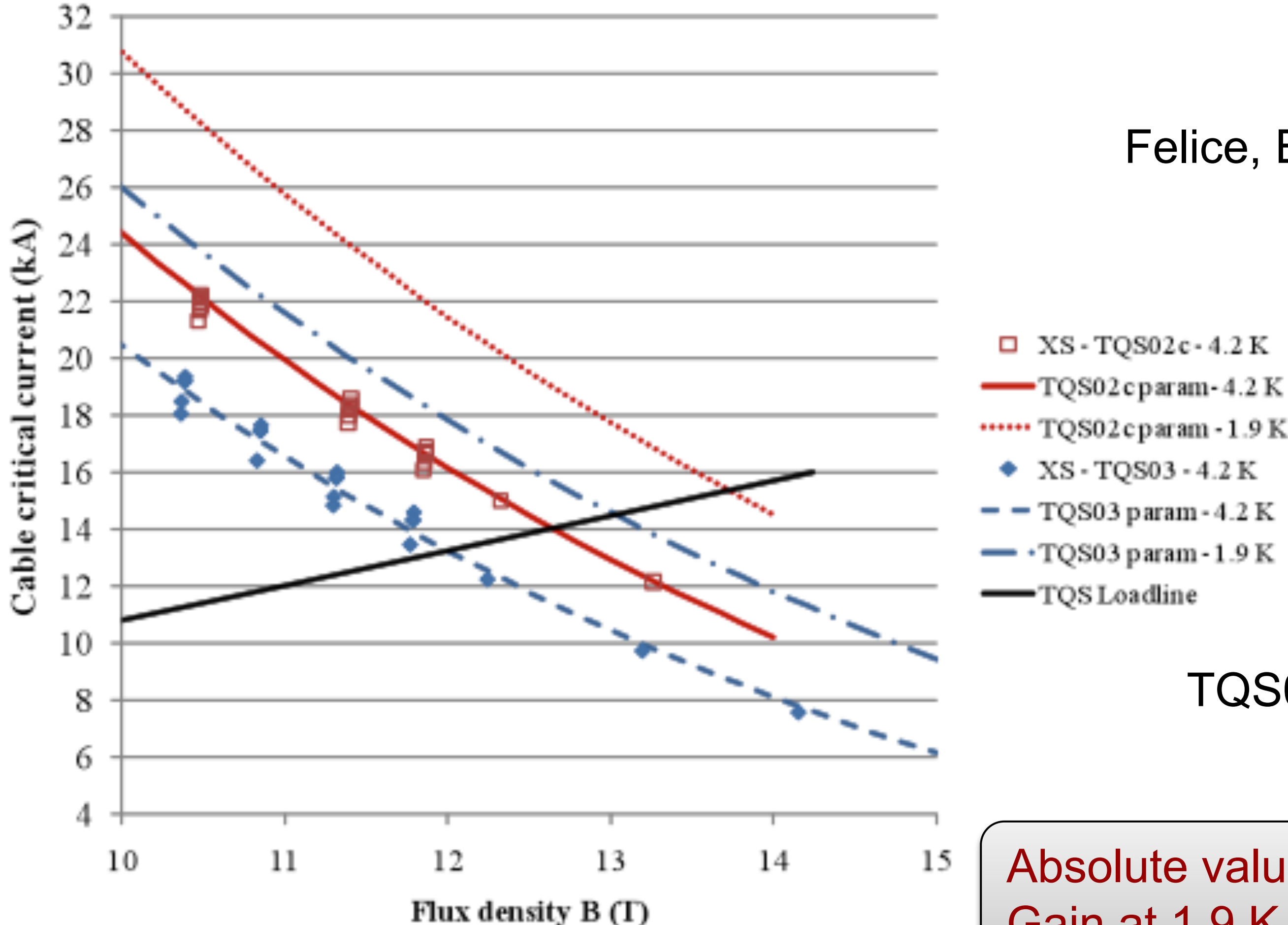
- Cable has additional – 0.085% axial strain compared to XS



What is the SS for the magnet? Closest strain match (barrel?)

Coil performance compared to XS parameterization

- XS scaling based on 4.2 K barrel data and estimated $T_{cm}(0)$ = 16.7 K



Felice, EUCAS 2009 TQ paper

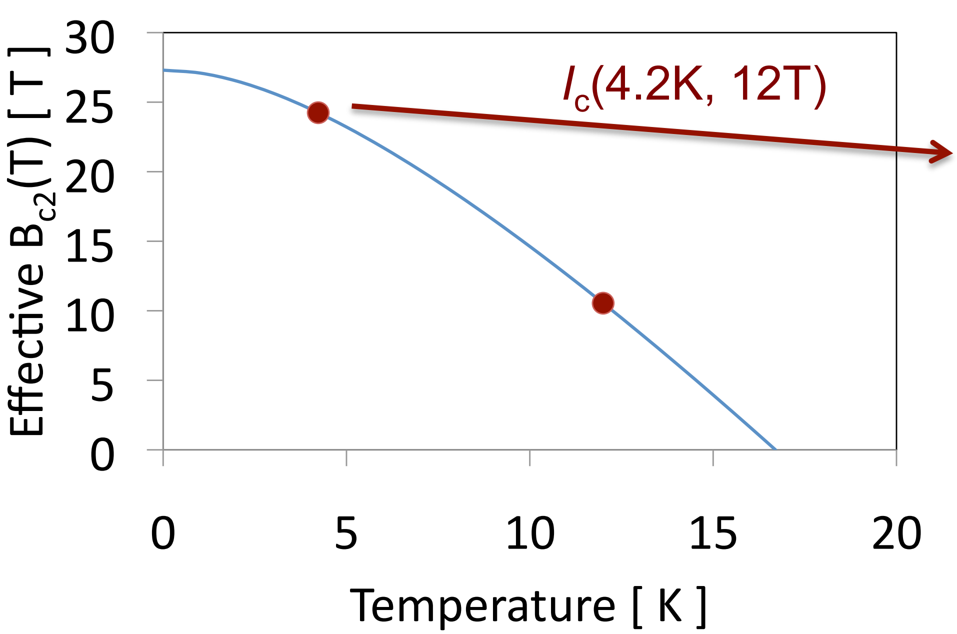
TQS03a = 93% at 4.3 K
= 93% at 1.9 K

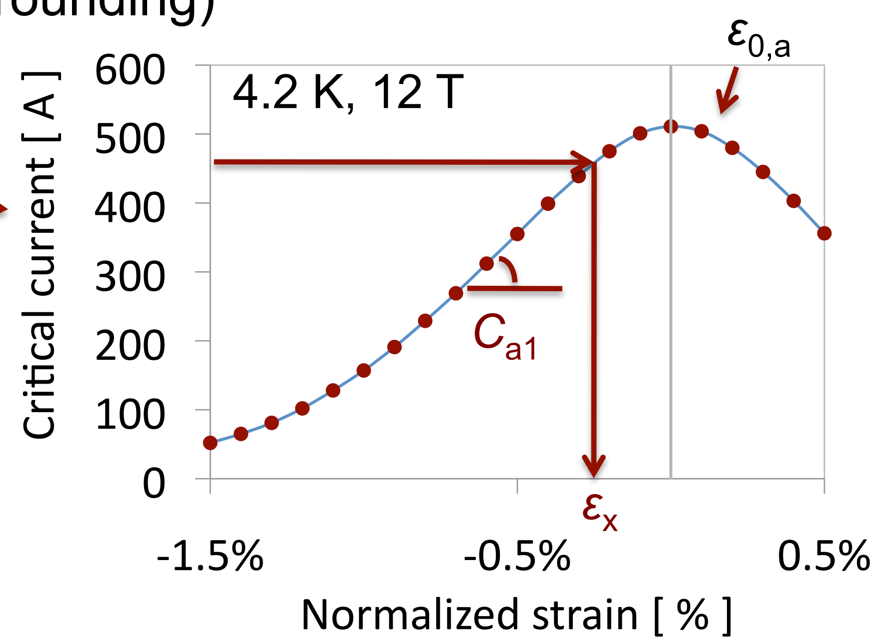
Absolute value SS unkown (strain)
Gain at 1.9 K is accurately *predicted*

Minimum required XS dataset

What is the minimum data required to fully parameterize an XS?

- $I_c(B)$ at 4.2 and, say, 12 K
  - Provides B_{c2}(4.2 K) and B_{c2}(12 K)
  - …and thus $B_{c2}(0,\varepsilon_x)$ and $T_c(0,\varepsilon_x)$
- $I_c(\varepsilon)$ at, say, 12 T and 4.2 K or 12 K
  - Provides ε_x, C_{a1} (slope), and $\varepsilon_{0,a}$ (peak rounding)





Only limited data is needed.

Summary

Emerging consensus on scaling relations

- Powers of temperature dependence still argued
- 'Latest greatest' 3D strain model needs 3D verification
- All is scaled axially, due to unknown 3D strain state of Nb_3Sn

Models can reasonably explain differences XS – cable – magnet

- Different strain state cable vs. XS very plausible
- 3D mechanical modeling lacking
- Limited SS measurements required to map $I_c(B,T,\varepsilon)$
- Absolute prediction (why 93%?) remains inaccessible (3D strain)
- Relative changes can be truly predictive